International Journal on Emerging Technologies **10**(4): 145-152(2019)

ISSN No. (Print): 0975-8364
ISSN No. (Online): 2249-3255

Modified Differential Transform Method to Study Reaction Diffusion Mechanism of Carcinogenic Polycyclic Aromatic Hydrocarbon in Mammalian Cell Including Perinuclear Membrane

Khushbu D. Patel[1] and D.C. Joshi[2]

[1]Assistant Professor, Department of Mathematics,
Government Engineering College, Valsad-396001 (Gujarat), India.
[2]Professor, Department of Mathematics,
Veer Narmad South Gujarat University, Surat-7 (Gujarat), India.

(Corresponding author: Khushbu D. Patel)

(Received 30 August 2019, Revised 25 October 2019, Accepted 05 November 2019)
(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: Polycyclic aromatic hydrocarbons are the potent atmospheric pollutants produced due to incomplete combustion of organic substances. BPDE (Benzo [a] pyrene) is one of the reactive carcinogenic compounds of PAHs which react with genetic material DNA of the cell and may turn into tumor or cancer. In this paper we study the mathematical modeling of the reaction diffusion mechanism of these toxic compounds in mammalian cells. In this paper, compartmental modeling approach have been used with the inclusion of perinuclear space. To study this model, we use Modified Differential Transform Method. Also, the obtained results are compared with results obtained from Differential Transform Method and Numerical Solution to prove the accuracy and efficiency of this method.

Keywords: Carcinogenic, Cell, Compartment Model, Modified differential transform method, Pade' approximate.

I. INTRODUCTION

The Mathematical Modelling of Intra-cellular reaction and diffusion mechanism is a challenging task due to complex cell architecture. Schematically, a mammalian cell is composed by cell membrane, cytoplasm (which contains many cell organelles like mitochondria, endo-plasmic reticulum, golgi apparatus, etc.) nuclear envelope, perinuclear space, nuclear membrane and nucleus which contain DNA [1].

Many researchers had developed mathematical models to study Reaction Diffusion Mechanism of Carcinogenic Polycyclic Aromatic Hydrocarbon (PAHs) in Mammalian Cell. In Chaudhry *et. al.*, (2009) [6] presented a mathematical model by using homogenization technique to study the behavior of lipophilic toxic compound in different cell geometry to overcome the difficulty caused by complex and heterogeneous structure of cytoplasm. In Dreij *et. al.,* (2011) [12] develop a mathematical model on diffusion-reaction mechanism of lipophilic compound to discuss the effect of various parameters like cell architecture, distribution of enzymes in the metabolism and hydrophobic nature of toxic compound. In Chaudhry *et. al.,* (2012) [8] to study the impact of cell geometry, develops mathematical modeling of reaction and diffusion in the cell. Then, Dreij *et al*., [11], proposed a mathematical model using homogenization technique to study the intracellular dynamics of PAH De and role of GST in protecting DNA from damage. Chaudhry *et al*., (2012) [7] presented model to study reaction and diffusion mechanism in cell with homogenization approach which include both volume and surface reaction. Chaudhry and Hanke (2014) [5], using nonstandard compartment model presented the numerical solution of system of PDE describing the reaction diffusion mechanism in cell. Qaiser *et al*., (2015) [19], Zainab *et. al*., (2014) [21] studied the drug diffusion and reaction process in the cell using homogenization by 2D axi symmetric mathematical model with spherical and non-spherical cell including nuclear envelop. Noor *et. al*., (2015) [17] used the compartmental modeling approach by replacing the system of PDEs with system of ODEs to study cellular exposure of reaction and diffusion mechanism of carcinogenic compound by including perinuclear space. Chaudhry *et al.,* (2012) [7] apply differential transform method to solve the system of ordinary differential equation developed from system of PDEs to study the reaction diffusion mechanism of PAHs in cell and compare the obtain result with numerical solution. Also, Patel (2017) [18] apply Differential Transform method to study compartment based model involve with reaction diffusion mechanism in cell including perinuclear space. But in this paper, we study compartment-based model involve with reaction diffusion mechanism in cell by Modified differential transform Method. This method gives the better solution than the usual differential Transform method. Also, we compare the obtained solution with numerical solution which concludes the accuracy and efficiency of the proposed method.

II. MATHEMATICAL FORMULATION

In this Paper, we are considering here four subdomains for our compartmental model namely extracellular medium, cytoplasm, perinuclear space and nucleus. Extracellular medium is the outside environment of the cell which contains water. Cytoplasm is that segment of cell between cell membrane and nuclear envelope that include cellular organelles and cytosol. Nuclear envelope is a barrier which monitors the import and export of molecules in and outside the nucleus. Nucleus carries ancestral material and is encompassed by nuclear membrane [2].

BPDE (Benzo pyrene diol epoxide) is one of the carcinogenic chemical compounds of PAHs is denoted as B_P in this mathematical system. BPDE undergo hydrolysis process within and outside the cell where water is available to form tetrols (BPT). Here tetrols are denoted as T_P. So, B_P reacts with water to yield T_P in extracellular medium. In this Model, there is no reaction taking place in membranes. When B_P and T_P come to the second compartment (cytoplasm), B_P reacts with water to yield T_P (tetrols), secondly B_P reacts with glutathione transferees to yield G_C (glutathione conjugate). Again, B_P and T_P come to the perinuclear space by diffusing through nuclear envelope and here B_P undergoes hydrolysis process to yield T_P. Lastly, when T_P and B_P reach the nucleus by diffusion where B_P reacts with water to yield T_P and B_P reacts with DNA resulting in D_A (DNA adduct) thereby engender toxicity, tumor or cancer. Thus, our compartmental modeling technique provides a tool to investigate the fate of carcinogenic compounds in mammalian cells. An index is added to distinguish the concentrations between the different compartments. For example, we denote B_P in first compartment (extracellular medium) by B_{P_1}. Fig. 1 demonstrates the compartmental system in and outside the mammalian cell.

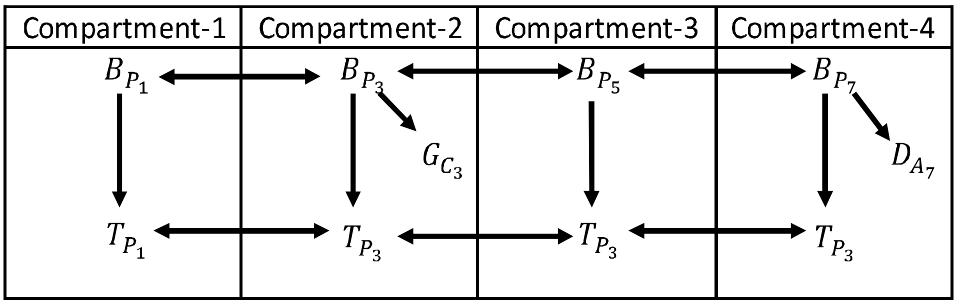

Fig. 1. Reaction-Diffusion process for four compartments.

Table 1: Notations and Process representation.

BPDE (Benzo pyrene diol epoxide)	B_P
BPT (Benzo pyrene Tetrol)	T_P
GSH Conjugate	G_C
DNA Adducts	D_A
Diffusion	$\longleftrightarrow$
Reaction	$\longrightarrow$

Set of ordinary differential equations obtained from compartment modeling is given as follows [7, 6, 11, 17, 18].

Compartment 1: (Extracellular Medium)

$$B_P \xrightarrow{k_{T_P}} T_P$$

$$\frac{d}{dt}B_{P_1} = \frac{DA_1}{V_1 K_{P,B_P}\delta}\left(\frac{B_{P_3}}{\sigma_{B_P}} - B_{P_1}\right) - k_{T_P}B_{P_1} \tag{1}$$

$$\frac{d}{dt}T_{P_1} = \frac{DA_1}{V_1 K_{P,T_P}\delta}\left(\frac{T_{P_3}}{\sigma_{T_P}} - T_{P_1}\right) + k_{T_P}B_{P_1} \tag{2}$$

Compartment 2: (Cytoplasm)

$$B_P \xrightarrow{k_{T_P}} T_P$$

$$B_P \xrightarrow{k_{G_C}} G_C$$

$$\frac{d}{dt}B_{P_3} = \frac{DA_1}{V_2 K_{P,B_P}\delta}\left(B_{P_1} - \frac{B_{P_3}}{\sigma_{B_P}}\right) + \frac{DA_2}{V_2 K_{P,B_P}\delta}\left(B_{P_5} - \frac{B_{P_3}}{\sigma_{B_P}}\right) - \left(\frac{k_{T_P}+k_{G_C}}{\sigma_{B_P}}\right)B_{P_3} \tag{3}$$

$$\frac{d}{dt}T_{P_3} = \frac{DA_1}{V_2 K_{P,T_P}\delta}\left(T_{P_1} - \frac{T_{P_3}}{\sigma_{T_P}}\right) + \frac{DA_2}{V_2 K_{P,T_P}\delta}\left(T_{P_5} - \frac{T_{P_3}}{\sigma_{T_P}}\right) + \frac{1}{V_2}\left(\frac{k_{T_P}}{\sigma_{B_P}}\right)B_{P_3} \tag{4}$$

$$\frac{d}{dt}G_{C_3} = \frac{k_{G_C}}{\sigma_{B_P}}B_{P_3} \tag{5}$$

Compartment 3: (Perinuclear Space)

$$B_P \xrightarrow{k_{T_P}} T_P$$

$$\frac{d}{dt}B_{P_5} = \frac{DA_2}{V_3 K_{P,B_P}\delta}\left(\frac{B_{P_3}}{\sigma_{B_P}} - B_{P_5}\right) + \frac{DA_3}{V_3 K_{P,B_P}\delta}\left(B_{P_7} - B_{P_5}\right) - k_{T_P}B_{P_5} \tag{6}$$

$$\frac{d}{dt}T_{P_5} = \frac{DA_2}{V_3 K_{P,T_P}\delta}\left(\frac{T_{P_3}}{\sigma_{T_P}} - T_{P_5}\right) + \frac{DA_3}{V_3 K_{P,T_P}\delta}\left(T_{P_7} - T_{P_5}\right) + k_{T_P}B_{P_5} \tag{7}$$

Compartment 4: (Nucleus)

$$B_P \xrightarrow{k_{T_P}} T_P$$

$$B_P \xrightarrow{k_{D_A}} D_A$$

$$\frac{d}{dt}B_{P_7} = \frac{DA_3}{V_4 K_{P,B_P}\delta}\left(B_{P_5} - B_{P_7}\right) - \left(k_{T_P} + k_{D_A}\right)B_{P_7} \tag{8}$$

$$\frac{d}{dt}T_{P_7} = \frac{DA_3}{V_4 K_{P,T_P}\delta}\left(T_{P_5} - T_{P_7}\right) + k_{T_P}B_{P_7} \tag{9}$$

$\frac{d}{dt}D_{A_7} = k_{D_A}B_{P_7}$ (10)

Initial Condition

$B_{P_1} = 10{,}000; T_{P_1} = B_{P_3} = T_{P_3} = G_{C_3} = B_{P_5} = T_{P_5} = B_{P_7} = T_{P_7} = D_{A_7} = 0$ (11)

III. BASIC IDEA OF DIFFERENTIAL TRANSFORM METHOD (DTM)

In this section, we discussed about the basic definitions and operation properties of differential transform method [1, 4, 9, 14, 15,16].

The differential transformation $F(k)$of a function $f(t)$is defined as follows:

$F(k) = \frac{1}{k!}\left[\frac{d^k f(t)}{dt^k}\right]_{t=0}$ (12)

In above equation $f(t)$ is the original function and $F(k)$ is the transformed function, which is called T-function. Also, inverse Differential transform of $F(k)$ is defined as

$f(t) = \sum_{k=0}^{\infty} t^k F(k)$ (13)

Table 2: Some operational properties of DTM.

Function	Differential Transform
$f(t) = g(t) \pm h(t)$	$F(k) = G(k) \pm H(k)$
$f(t) = cg(t)$, where c is any constant	$F(k) = cG(k)$
$f(t) = \frac{dg(t)}{dt}$	$F(k) = (k+1)G(k+1)$
$f(t) = \frac{d^m g(t)}{dt^m}$	$F(k) = (k+1)(k+2)\ldots.(k+m)G(k+m)$
$f(t) = t^m$	$F(k) = \delta(k-m) = \begin{cases} 1, \text{if } k = m \\ 0, \text{if } k \neq m \end{cases}$
$f(t) = g(t)h(t)$	$F(k) = \sum_{r=0}^{k} H(r)G(k-r)$
$f(t) = e^{mt}$	$F(k) = \frac{m^k}{k!}$
$f(t) = \sin(\omega t + \alpha)$	$F(k) = \frac{\omega^k}{k!}\sin\left(\frac{\pi k}{2} + \alpha\right)$
$f(t) = \cos(\omega t + \alpha)$	$F(k) = \frac{\omega^k}{k!}\cos\left(\frac{\pi k}{2} + \alpha\right)$

IV. THE PADE' APPROXIMANTS

Some techniques exist to increase the convergence of given power series. Among them the pade approximants is widely applied.

Suppose we have power series expansion of a function $f(t)$ given by

$f(t) = \sum_{i=0}^{\infty} a_i t^i$ (14)

Then the Pade approximants to $f(t)$ of order [L, M] which we denote by $\left[\frac{L}{M}\right]_u (t)$ is defined as Eqn. [12]

$\left[\frac{L}{M}\right]_u (t) = \frac{P_L(t)}{Q_M(t)} = \frac{p_0 + p_1 t + \cdots + p_L t^L}{1 + q_1 t + \cdots + q_M t^M}$ (15)

where we considered $q_0 = 1$ and the numerator and denominator have no common factors. Construct the numerator and denominator in Eqn. (15) so that $f(t)$ and $\left[\frac{L}{M}\right]_u (t)$ and their derivatives agree at $t = 0$ up to $L + M$.That is,

$f(t) - \left[\frac{L}{M}\right]_u (t) = O(t^{L+M+1})$ (16)

From Eqn. (16), we get,

$f(t)\sum_{n=0}^{M} q_n t^n - \sum_{n=0}^{L} p_n t^n = O(t^{L+M+1})$ (17)

From Eqn. (17) yield the following set of equations

$$
\begin{aligned}
&a_L q_1 + \cdots + a_{L-M+1} q_M = -a_{L+1}, \\
&a_{L+1} q_1 + \cdots a_{L-M+2} q_M = -a_{L+2}, \\
&\vdots \\
&a_{L+M-1} q_1 + \cdots a_L q_M = -a_{L+M},
\end{aligned} \qquad (18)
$$

$$
\begin{aligned}
&p_0 = a_0 \\
&p_1 = a_1 + a_0 q_1 \\
&\vdots \\
&p_L = a_L + a_{L-1} q_1 + \cdots + a_0 q_L
\end{aligned} \qquad (19)
$$

From Eqns. (18) and (19), we calculate all the coefficients q_n, $1 \leq n \leq M$ and p_n , $0 \leq n \leq L$. Note that, for a fixed value of L+M+1 error Eqn. (17) is smallest when the numerator and denominator of Eqn. (16) have the same degree or when the numerator has one degree higher than the denominator.

V. MODIFIED DIFFERENTIAL TRANSFORM METHOD (MDTM)

Modified Differential Transform Method (MDTM) is the combination of Differential Transform Method (DTM), Laplace Transform and Pade Approximants. This method can be explained as follows: [3, 10, 13, 20]

(1) Find the power series solution of the given problem by applying DTM.

(2) Replace s by $1/t$ in the resulting equation.

(3) Convert the transformed series into the meromorphic function by forming its pade approximant of order [L/M]. N and M are arbitrarily chosen, but they should be smaller than the order of the power series. In this step, the pade approximant extends the domain of truncated series solution to obtain better accuracy and convergence.
(4) After that, replace t by $1/s$.
(5) Finally, using the inverse Laplace transform we obtain the exact or approximate solution.

VI. SOLUTION BY MDTM

By Applying Differential Transform Method to above system of ordinary differential Eqns. (1) to (10) we get following system of recursive formula.

Compartment 1: (Extracellular Medium)

$$B_{P_1}(k+1)=\frac{1}{(k+1)}\left[\frac{DA_1}{V_1K_{P,B_P}\delta}\left(\frac{B_{P_3}(k)}{\sigma_{B_P}}-B_{P_1}(k)\right)-k_{T_P}B_{P_1}(k)\right] \tag{20}$$

$$T_{P_1}(k+1)=\frac{1}{(k+1)}\left[\frac{DA_1}{V_1K_{P,T_P}\delta}\left(\frac{T_{P_3}(k)}{\sigma_{T_P}}-T_{P_1}(k)\right)+k_{T_P}B_{P_1}(k)\right] \tag{21}$$

Compartment 2: (Cytoplasm)

$$B_{P_3}(k+1)=\frac{1}{(k+1)}\left[\frac{DA_1}{V_2K_{P,B_P}\delta}\left(B_{P_1}(k)-\frac{B_{P_3}(k)}{\sigma_{B_P}}\right)+\frac{DA_2}{V_2K_{P,B_P}\delta}\left(B_{P_5}(k)-\frac{B_{P_3}(k)}{\sigma_{B_P}}\right)-\left(\frac{k_{T_P}+k_{G_C}}{\sigma_{B_P}}\right)B_{P_3}(k)\right] \tag{22}$$

$$T_{P_3}(k+1)=\frac{1}{(k+1)}\left[\frac{DA_1}{V_2K_{P,T_P}\delta}\left(T_{P_1}(k)-\frac{T_{P_3}(k)}{\sigma_{T_P}}\right)+\frac{DA_2}{V_2K_{P,T_P}\delta}\left(T_{P_5}(k)-\frac{T_{P_3}(k)}{\sigma_{T_P}}\right)+\frac{1}{V_2}\left(\frac{k_{T_P}}{\sigma_{B_P}}\right)B_{P_3}(k)\right] \tag{23}$$

$$G_{C_3}(k+1)=\frac{1}{(k+1)}\left[\frac{k_{G_C}}{\sigma_{B_P}}B_{P_3}(k)\right] \tag{24}$$

Compartment 3: (Perinuclear Space)

$$B_{P_5}(k+1)=\frac{1}{(k+1)}\left[\frac{DA_2}{V_3K_{P,B_P}\delta}\left(\frac{B_{P_3}(k)}{\sigma_{B_P}}-B_{P_5}(k)\right)+\frac{DA_3}{V_3K_{P,B_P}\delta}\left(B_{P_7}(k)-B_{P_5}(k)\right)-k_{T_P}B_{P_5}(k)\right] \tag{25}$$

$$T_{P_5}(k+1)=\frac{1}{(k+1)}\left[\frac{DA_2}{V_3K_{P,T_P}\delta}\left(\frac{T_{P_3}(k)}{\sigma_{T_P}}-T_{P_5}(k)\right)+\frac{DA_3}{V_3K_{P,T_P}\delta}\left(T_{P_7}(k)-T_{P_5}(k)\right)+k_{T_P}B_{P_5}(k)\right] \tag{26}$$

Compartment 4: (Nucleus)

$$B_{P_7}(k+1)=\frac{1}{(k+1)}\left[\frac{DA_3}{V_4K_{P,B_P}\delta}\left(B_{P_5}(k)-B_{P_7}(k)\right)-\left(k_{T_P}+k_{D_A}\right)B_{P_7}(k)\right] \tag{27}$$

$$T_{P_7}(k+1)=\frac{1}{(k+1)}\left[\frac{DA_3}{V_4K_{P,T_P}\delta}\left(T_{P_5}(k)-T_{P_7}(k)\right)+k_{T_P}B_{P_7}(k)\right] \tag{28}$$

$$D_{A_7}(k+1)=\frac{1}{(k+1)}k_{D_A}B_{P_7}(k) \tag{29}$$

Also, by applying Differential Transform Method to Initial Condition (2.11), we have,

$$B_{P_1}(0)=10{,}000;\ T_{P_1}(0)=B_{P_3}(0)=T_{P_3}(0)=G_{C_3}(0)=B_{P_5}(0)=T_{P_5}(0)=B_{P_7}(0)=T_{P_7}(0)=D_{A_7}(0)=0 \tag{30}$$

The Geometry constant and parameter values used in this model are given in Table 3 and 4 [1, 17, 19, 21].

Table 3: Geometry constant for Model.

Constants	Value	Units
Volume of extracellular Medium (V_1)	663666.67	μm^3
Volume of cytoplasm (V_2)	2690.9662	μm^3
Volume of perinuclear space (V_3)	9.033755368	μm^3
Volume of nucleus (V_4)	3000	μm^3
Area of cell membrane (A_1)	1005.9235	μm^2
Area of nuclear envelope (A_2)	222.2385791	μm^2
Area of nuclear membrane (A_3)	217.8977843	μm^2

Table 4: Parameter values used for Model.

Symbol	Constant	Value
k_{T_P}	T_P(Tetrols) formation rate	0.0077
k_{D_A}	D_A(DNA adduct) formation rate	0.0062
k_{G_C}	G_C(GSH conjugate) formation rate in homogenized cytoplasm	0.242908
k_{T_P}	T_P(Tetrols) formation rate in homogenized cytoplasm	0.005744
D	Diffusion in membrane	10^{-12}
σ_{B_P}	Scaling Factor for B_P	212.4
σ_{T_P}	Scaling Factor for T_P	31.3
K_{p,B_P}	Partition Coefficient for BPDE	0.0012
K_{p,T_P}	Partition Coefficient of BPT	0.0083

By Taking $k=0,1,2,3,4,\ldots.$ in above recursive relation Eqns. (20)-(29) and using initial transform coefficients Eqn. (30), we get the following solution of given compartment model,

$B_{P_1} = 10000 + (-77t) + (0.29645t^2) + (-0.000760889t^3) + (0.00000146471t^4) + (-2.25565 \times 10^{-9})t^5 + (2.89476 \times 10^{-12})t^6 + (-3.18423 \times 10^{-15})t^7 + (3.06482 \times 10^{-18})t^8 + (-2.62213 \times 10^{-21})t^9 + \cdots$ (31)

$T_{P_1} = 0 + 77t + (-0.29645)t^2 + (0.000760888)t^3 + (-0.00000146471)t^4 + (2.25565 \times 10^{-9})t^5 + (-2.89476 \times 10^{-12})t^6 + (3.18423 \times 10^{-15})t^7 + (-3.06482 \times 10^{-18})t^8 + (2.62213 \times 10^{-21})t^9 + \cdots$ (32)

$B_{P_3} = 0 + 0.00155756t + (-0.00000691553)t^2 + (1.81113 \times 10^{-8})t^3 + (-3.49709 \times 10^{-11})t^4 + (5.38803 \times 10^{-14})t^5 + (-6.91513 \times 10^{-17})t^6 + (7.60673 \times 10^{-20})t^7 + (-7.32149 \times 10^{-23})t^8 + (6.26394 \times 10^{-26})t^9 + \cdots$ (33)

$T_{P_3} = 0 + (0 \times t) + (8.66991 \times 10^{-7})t^2 + (-2.22528 \times 10^{-9})t^3 + (4.28367 \times 10^{-12})t^4 + (-6.59685 \times 10^{-15})t^5 + (8.46596 \times 10^{-18})t^6 + (-9.31256 \times 10^{-21})t^7 + (8.96334 \times 10^{-24})t^8 + (-7.66863 \times 10^{-27})t^9 + \cdots$ (34)

$G_{C_3} = 0 + (0 \times t) + (8.90683 \times 10^{-7})t^2 + (-2.63641 \times 10^{-9})t^3 + (5.17842 \times 10^{-12})t^4 + (-7.99915 \times 10^{-15})t^5 + (1.02704 \times 10^{-17})t^6 + (-1.12982 \times 10^{-20})t^7 + (1.08747 \times 10^{-23})t^8 + (-9.30389 \times 10^{-27})t^9 + \cdots$ (35)

$B_{P_5} = 0 + (0 \times t) + (3.75856 \times 10^{-11})t^2 + (-2.07977 \times 10^{-13})t^3 + (6.19933 \times 10^{-16})t^4 + (-1.29477 \times 10^{-18})t^5 + (2.09939 \times 10^{-21})t^6 + (-2.79219 \times 10^{-24})t^7 + (3.15346 \times 10^{-27})t^8 + (-3.09769 \times 10^{-30})t^9 + \cdots$ (36)

$T_{P_5} = 0 + (0 \times t) + (0 \times t^2) + (1.10136 \times 10^{-13})t^3 + (-4.26743 \times 10^{-16})t^4 + (9.95459 \times 10^{-19})t^5 + (-1.71409 \times 10^{-21})t^6 + (2.36724 \times 10^{-24})t^7 + (-2.7434 \times 10^{-27})t^8 + (2.74595 \times 10^{-30})t^9 + \cdots$ (37)

$B_{P_7} = 0 + (0 \times t) + (0 \times t^2) + (3.79158 \times 10^{-19})t^3 + (-2.89111 \times 10^{-21})t^4 + (1.17896 \times 10^{-23})t^5 + (-3.38433 \times 10^{-6})t^6 + (7.62797 \times 10^{-29})t^7 + (-1.43099 \times 10^{-31})t^8 + (2.31613 \times 10^{-34})t^9 + \cdots$ (38)

$T_{P_7} = (8.50353 \times 10^{-22})t^4 + (-4.82574 \times 10^{-24})t^5 + (1.58559 \times 10^{-26})t^6 + (-3.8299 \times 10^{-29})t^7 + (7.47139 \times 10^{-32})t^8 + (-1.23763 \times 10^{-34})t^9 + \cdots$ (39)

$D_{A_7} = (5.87695 \times 10^{-22})t^4 + (-3.58497 \times 10^{-24})t^5 + (1.21826 \times 10^{-26})t^6 + (-2.99755 \times 10^{-29})t^7 + (5.91167 \times 10^{-32})t^8 + (-9.85793 \times 10^{-35})t^9 + \cdots$ (40)

We apply Laplace Transform to above equations and then for simplicity, substituting$s = \frac{1}{t}$ in obtained equations, we have following equations,

$L\{B_{P_1}\} = 10000t + (-77)t^2 + 0.5929t^3 - 0.00456533t^4 + 0.000035153t^5 - (2.70678 \times 10^{-7})t^6 + (2.08423 \times 10^{-9})t^7 - (1.60485 \times 10^{-11})t^8 + (1.23574 \times 10^{-13})t^9 - (9.51519 \times 10^{-16})t^{10} + \cdots$ (41)

$L\{T_{P_1}\} = 77.0t^2 - 0.5929t^3 + 0.00456533t^4 - 0.000035153t^5 + (2.70678 \times 10^{-7})t^6 - (2.08423 \times 10^{-9})t^7 + (1.60485 \times 10^{-11})t^8 - (1.23574 \times 10^{-13})t^9 + (9.51519 \times 10^{-16})t^{10} + \cdots$ (42)

$L\{B_{P_3}\} = 0.00155756t^2 - 0.0000138311t^3 + (1.08668 \times 10^{-7})t^4 - (8.39302 \times 10^{-10})t^5 + (6.46564 \times 10^{-12})t^6 - (4.97889 \times 10^{-14})t^7 + (3.83379 \times 10^{-16})t^8 - (2.95202 \times 10^{-18})t^9 + (2.27306 \times 10^{-20})t^{10}$ (43)

$L\{T_{P_3}\} = 0.00000173398t^3 - (1.33517 \times 10^{-8})t^4 + (1.02808 \times 10^{-10})t^5 - (7.91622 \times 10^{-13})t^6 + (6.09549 \times 10^{-15})t^7 - (4.69353 \times 10^{-17})t^8 + (3.61402 \times 10^{-19})t^9 - (2.78279 \times 10^{-21})t^{10} + \cdots$ (44)

$L\{G_{C_3}\} = 0.00000178137\, t^3 - (1.58185 \times 10^{-8})t^4 + (1.24282 \times 10^{-10})t^5 - (9.59898 \times 10^{-13})t^6 + (7.39469 \times 10^{-15})t^7 - (5.69429 \times 10^{-17})t^8 + (4.38468 \times 10^{-19})t^9 - (3.3762 \times 10^{-21})t^{10} + \cdots$ (45)

$L\{B_{P_5}\} = (7.51712 \times 10^{-11})t^3 - (1.24786 \times 10^{-12})t^4 + (1.48784 \times 10^{-14})t^5 - (1.55372 \times 10^{-16})t^6 + (1.51156 \times 10^{-18})t^7 - (1.40726 \times 10^{-20})t^8 + (1.27148 \times 10^{-22})t^9 - (1.12409 \times 10^{-24})t^{10} + \cdots$ (46)

$L\{T_{P_5}\} = (6.60816 \times 10^{-13})t^4 - (1.02418 \times 10^{-14})t^5 + (1.19455 \times 10^{-16})t^6 - (1.23414 \times 10^{-18})t^7 + (1.19309 \times 10^{-20})t^8 - (1.10614 \times 10^{-22})t^9 + (9.9645 \times 10^{-25})t^{10} + \cdots$ (47)

$L\{B_{P_7}\} = (2.27495 \times 10^{-18})t^4 - (6.93866 \times 10^{-20})t^5 + (1.41475 \times 10^{-21})t^6 - (2.43672 \times 10^{-23})t^7 + (3.8445 \times 10^{-25})t^8 - (5.76975 \times 10^{-27})t^9 + (8.40477 \times 10^{-29})t^{10} + \cdots$ (48)

$L\{T_{P_7}\} = (2.04085 \times 10^{-20})t^5 - (5.79089 \times 10^{-22})t^6 + (1.14162 \times 10^{-23})t^7 - (1.93027 \times 10^{-25})t^8 + (3.01246 \times 10^{-27})t^9 - (4.49111 \times 10^{-29})t^{10} + \cdots$ (49)

$L\{D_{A_7}\} = (1.41047 \times 10^{-20})t^5 - (4.30196 \times 10^{-22})t^6 + (8.77147 \times 10^{-24})t^7 - (1.51077 \times 10^{-25})t^8 + (2.38359 \times 10^{-27})t^9 - (3.57725 \times 10^{-29})t^{10} + \cdots$ (50)

The padé approximant [5/5] of all above equations and substituting $t = \frac{1}{s}$, we obtain [5/5] padé approximant in terms of s. Then By using the inverse Laplace transformation, we obtain the following solution of ode system (1) -(10)

$B_{P_1} =$
$10000\exp(-0.0077t) - \exp(-0.000299229t)\cos(0.00900918t)(0.000809925 - 0.00156737i) - \exp(-0.000299229t)\cos(0.00900918t)(0.000809925 + 0.00156737i) + \exp(-0.000299229t)\sin(0.00900918t)(0.00156737 + 0.000809925i) + \exp(-0.000299229t)\sin(0.00900918t)(0.00156737 - 0.000809925i) - \exp(0.0079761t)\cos(0.00764422t)(0.0002313 + 0.00078052i) - \exp(0.0079761t)\cos(0.00764422t)(0.0002313 - 0.00078052i) - \exp(0.0079761t)\sin(0.00764422t)(0.00078052 - 0.0002313i) - \exp(0.0079761t)\sin(0.00764422t)(0.00078052 + 0.0002313i)$ (51)

$$T_{P_1} = 10000\exp(2.1682e-8t) - 0.0183866\exp(0.00603802t) - 9999.99\exp(-0.0077t) - \exp(0.0034971t)\cos(0.00765012t)(0.00365881 - 0.000802464i) - \exp(0.0034971t)\cos(0.00765012t)(0.00365881 + 0.000802461i) - \exp(0.0034971t)\sin(0.00765012t)(0.000802464 + 0.00365881i) - \exp(0.0034971t)\sin(0.00765012t)(0.000802461 - 0.00365881i) \quad (52)$$

$$B_{P_3} = 0.238889\exp(-0.00117995t) - (8.83515\times10^{-7})\exp(0.00185046t) - 0.238888\exp(-0.0077t) - (3.2846\times10^{-7})\exp(0.000917071t)\cos(0.00607493t) + (2.53559\times10^{-7})\exp(0.000917071t)\sin(0.00607493t) \quad (53)$$

$$T_{P_3} = 8.9272\exp(0.0000125764t) - (1.64025\times10^{-7})\exp(-0.00962079t) - 8.95645\exp(-0.000012608t) + 0.00000335076\exp(-0.00649626t) + 0.0292431\exp(-0.00770009t) \quad (54)$$

$$G_{C_3} = 0.196086\exp\left((-5.72413\times10^{-8})t\right) - 0.231569\exp(-0.00117989t) + 0.0354822\exp(-0.0077t) - \exp(0.00814453t)\cos(0.00560404t)(4.48872e-9 + 7.22587e-9i) - \exp(0.00814453t)\cos(0.00560404t)(4.48872e-9 - 7.22582e-9i) - \exp(0.00814453t)\sin(0.00560404t)(7.22587e-9 - 4.48872e-9i) - \exp(0.00814453t)\sin(0.00560404t)(7.22582e-9 + 4.48872e-9i) \quad (55)$$

$$B_{P_5} = 0.00000176294\exp(-0.00118029t) - 0.00030069\exp(-0.00769089t) + 0.000298927\exp(-0.00772929t) + \exp(0.00603948t)\cos(0.00304507t)\left((1.01259\times10^{-11}) - (2.85518\times10^{-12})i\right) + \exp(0.00603948t)\cos(0.00304507t)\left((1.01259\times10^{-11}) - (2.85518\times10^{-12})i\right) - \exp(0.00603948t)\sin(0.00304507t)\left((2.85518\times10^{-12}) + (1.01259\times10^{-11})i\right) - \exp(0.00603948t)\sin(0.00304507t)\left((2.85518\times10^{-12}) - (1.01259\times10^{-11})i\right) \quad (56)$$

$$T_{P_5} = 0.000150346\exp(-0.0076709t) - 0.00000456283\exp(-0.0015378t) - 0.000147267\exp(-0.00774823t) + 0.00000148445\exp(0.000593486t)\cos(0.000523073t) + 0.00000827334\exp(0.000593486t)\sin(0.000523073t) \quad (57)$$

$$B_{P_7} = (4.19729\times10^{-12})\exp(-0.00118144t) - (4.66825\times10^{-12})\exp(-0.0139t) + (2.14411\times10^{-17})\exp(0.00749396t) - (4.28405\times10^{-10})\exp(-0.00764399t) + (4.28876\times10^{-10})\exp(-0.00777534t) \quad (58)$$

$$T_{P_7} = (1.55433\times10^{-11})\exp(0.00106261t) - (2.21243\times10^{-11})\exp(-0.0000684791t) + (2.56867\times10^{-12})\exp(-0.0139067t) + \exp(-0.00773116t)\cos(0.000465637t)\left((2.00615\times10^{-12}) - (5.23047\times10^{-11})i\right) + \exp(-0.00773116t)\cos(0.000465637t)\left((2.00615\times10^{-12}) + (5.23047\times10^{-11})i\right) + \exp(-0.00773116t)\sin(0.000465637t)\left((5.23047\times10^{-11}) + (2.00615\times10^{-12})i\right) + \exp(-0.00773116t)\sin(0.000465637t)\left((5.23047\times10^{-11}) - (2.00615\times10^{-12})i\right) \quad (59)$$

$$D_{A_7} = (1.36565\times10^{-11})\exp(0.0000262425t) - (2.12789\times10^{-11})\exp(-0.00121488t) - (1.24301\times10^{-10})\exp(-0.00788489t) + (2.08353\times10^{-12})\exp(-0.0138994t) + (1.2984\times10^{-10})\exp(-0.00752735t) \quad (60)$$

VII. RESULT AND DISCUSSION

In this Model we study the formation and degradation of PAH DEs in the different compartment of the cell for 1200 seconds. Fig. 2 shows the formation and degradation of PAH DEs in extracellular, cytoplasm, perinuclear space and nucleus compartment. In this model we find the analytic approximate solution for mammalian V79 cell by considering perinuclear space using modified differential transform method. In Fig. 2, B_{P_1} shows the degradation of BPDE in extracellular medium which approaches to zero after 600 seconds, T_{P_1} shows the formation of BPT in extracellular compartment and reaches the steady sate condition after 600 seconds, G_{C_3} shows the formation of GSH conjugate and reaches the steady sate condition after 1200 seconds, D_{A_7} shows the formation of DNA adduct in the nucleus and reaches the steady sate condition after 1200 seconds.

Comparison of obtained analytical solution by MDTM with DTM and RK4:

In this Paper, the analytical approximate solution obtained by using MDTM are compared with the numerical solution obtained by using Runge Kutta fourth order method and usual DTM for 1200 second. From the Fig. 3, it is clear that solution obtained from MDTM is perfectly matched with numerical solution (solution obtained by Runge Kutta fourth order), while the obtained solutions from DTM is valid for small values of t. Therefore, we can conclude that the solutions obtained by MDTM are more accurate than solutions obtained from DTM.

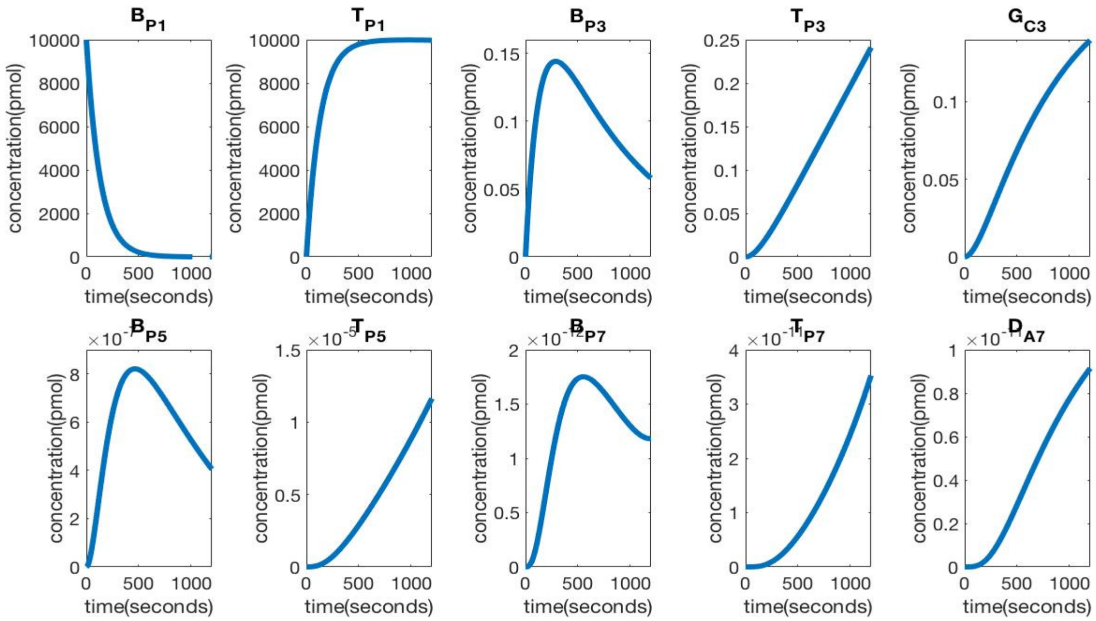

Fig. 2. Formation and Degradation of BPDE, BPT, GSH Conjugate.

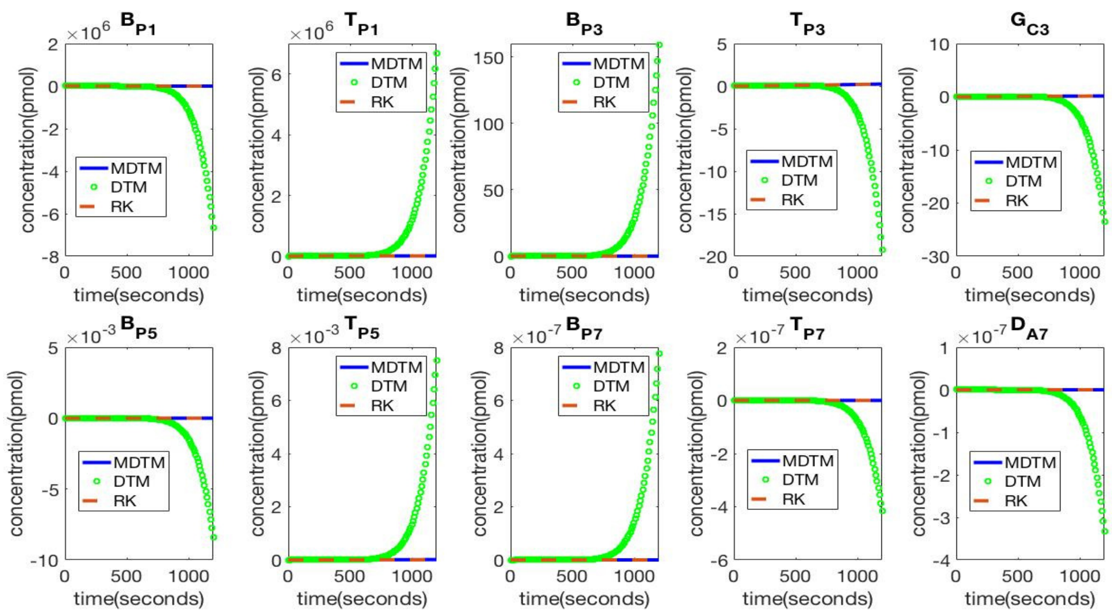

Fig. 3. Comparison of solution obtained by MDTM, DTM and RK fourth order for BPDE, BPT, GSH Conjugate.

VIII. CONCLUSION AND FUTURE SCOPE

In this paper, we present Modified Differential Transform Method as a combination of Differential Transform, Laplace transforms and Pade approximant to study compartment-based models of reaction and diffusion mechanism of carcinogenic polycyclic aromatic hydrocarbons in mammalian cell including perinuclear space. The method has been applied directly without requiring linearization, discretization or perturbation. Additionally, this Method does not require a perturbation parameter to work and it does not generate secular terms (noise terms) as other semi analytical methods like Homotopy Peturbation Method, Adomian Decomposition Method, or Variational Iteration Method.

Comparison between the solutions obtained by the DTM and MDTM with numerical solution (fourth-order Runge-Kutta) remarked that the accuracy of MDTM is very good.

MDTM is simple, easy to use, and is readily adaptable for computer implementation. So, further research should be performed to solve wider class of linear and highly nonlinear dynamic models based on ordinary differential equations as well as partial differential equations.

ACKNOWLEDGEMENT

The authors thank referees for fruitful comments and suggestions for revising the manuscript.

Conflict of Interest. The authors declare that there is no conflict of interest of any sort on this research.

REFERENCES

[1]. Arikoglu, A., & Ozkol, I. (2005). Solution of boundary value problems for integro-differential equations by using differential transform method. *Applied Mathematics and Computation*, *168*(2), 1145-1158.

[2]. Baker, G. A. (1975). *Essentials of Padé approximants*. academic press.

[3]. Benhammouda, B., Vazquez-Leal, H., & Sarmiento-Reyes, A. (2014). Modified reduced differential transform method for partial differential-algebraic equations. *Journal of applied mathematics*, *2014*.

[4]. Bert, C. W., & Zeng, H. (2004). Analysis of axial vibration of compound bars by differential transformation method. *Journal of Sound and Vibration*, *275*(3-5), 641-647.

[5]. Chaudhry, Q. A., & Hanke, M. (2014). On the numerical approximation of reaction and Diffusion Mechanism in a cell: A Non-Standard Compartment Model. *Pakistan Journal of Science*, *66*(4). 311-316.

[6]. Chaudhry, Q. A., Hanke, M., & Morgenstern, R. (2009). On the numerical approximation of drug diffusion in complex cell geometry. In *Proceedings of the 7th International Conference on Frontiers of Information Technology* (p. 17). ACM.

[7]. Chaudhry, Q. A., Hanke, M., Dreij, K., & Morgenstern, R. (2012). Mathematical modeling of reaction and diffusion systems in a cell including surface reactions on the cytoplasmic membranes. *Report TRITA-NA-2012:4, KTH Royal Inst. of Technology, Stockholm*.

[8]. Chaudhry, Q. A., Morgenstern, R., Hanke, M., & Dreij, K. (2012). Influence of biological cell geometry on reaction and diffusion simulation. Report TRITA-NA-2012:2, KTH Royal Inst. of Technology, Stockholm.

[9]. Chen, S. S. (2004). Application of the differential transformation method to a non-linear conservative system. *Applied Mathematics and Computation*, *154*(2), 431-441.

[10]. Dehghan, M., Hamidi, A., & Shakourifar, M. (2007). The solution of coupled Burgers' equations using Adomian–Pade technique. *Applied Mathematics and Computation*, *189*(2), 1034-1047.

[11]. Dreij, K., Chaudhry, Q. A., Jernström, B., Hanke, M., & Morgenstern, R. (2012). In silico modeling of intracellular diffusion and reaction of benzo [a] pyrene diol epoxide.

[12]. Dreij, K., Chaudhry, Q. A., Jernström, B., Morgenstern, R., & Hanke, M. (2011). A method for efficient calculation of diffusion and reactions of lipophilic compounds in complex cell geometry. *PLoS One*, *6*(8), e23128.

[13]. Ibrahim, S. F. M., & Ismail, S. M. (2013). A new modification of the differential transform method for a SIRC influenza model. *International Journal of Computer Applications*, *69*(19), 8-15.

[14]. Kuo, B. L., (2004). Thermal boundary-layer problems in a semi-infinite flat plate by the differential transformation method. Applied Mathematics and Computation, 150(2), 303-320.

[15]. Kuo, B. L. (2005). Heat transfer analysis for the Falkner–Skan wedge flow by the differential transformation method. *International Journal of Heat and Mass Transfer*, *48*(23-24), 5036-5046.

[16]. Malik, M. O. I. N. U. D. D. I. N., & Allali, M. O. H. A. M. E. D. (2000). Characteristic equations of rectangular plates by differential transformation. *Journal of sound and vibration*, *233*(2), 359-366.

[17]. Noor, A., Chaudhry, Q. A., Ikhlaq, A., Shuaib, A., & Abbas, A. (2015). Compartmental System of Reaction and Diffusion Mechanism of Carcinogenic Polycyclic Aromatic Hydrocarbons in Mammalian Cell. *Journal of Faculty of Engineering & Technology*, *22*(1), 53-62.

[18]. Patel, Y. (2017). Application of differential transform method to compartment modelling, (Ph.d. Thesis), SVNIT, Surat, India.

[19]. Qaiser, M., Chaudhry, Q. A., & Zainab, S. (2015). A simplified spherical cell model with the inclusion of nuclear envelope for the reaction-diffusion mechanism. *Science International*, *27*(2), 1121-1123.

[20]. Rashidi, M. M., Erfani, E., Bég, O. A., & Ghosh, S. K. (2011). Modified Differential Transform Method (DTM) simulation of hydromagnetic multi-physical flow phenomena from a rotating disk. *World Journal of Mechanics*, *1*(05), 217-230.

[21]. Zainab, S., Chaudhry, Q. A., & Qaiser, M. (2014). Inclusion of nuclear envelope in non-spherical cellular geometry. *space [m]*, *30*, 10-19.

How to cite this article: Patel, K. D. and Joshi, D. C. (2019). Modified Differential Transform Method to Study Reaction Diffusion Mechanism of Carcinogenic Polycyclic Aromatic Hydrocarbon In Mammalian Cell Including Perinuclear Membrane. *International Journal on Emerging Technologies*, *10*(4): 145–152.